Impedance Spectroscopy & Applications

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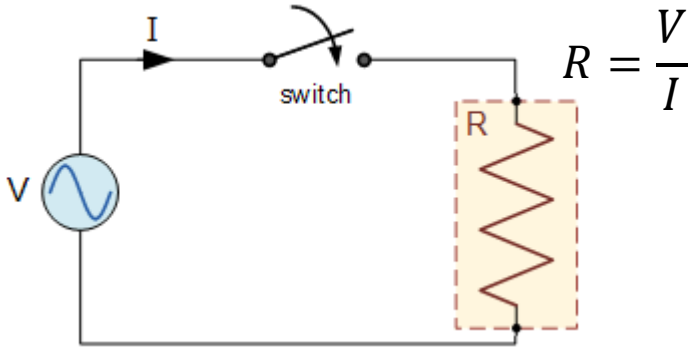
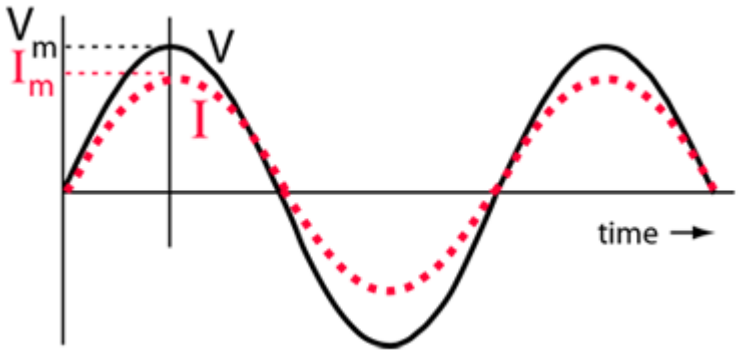
Resistance

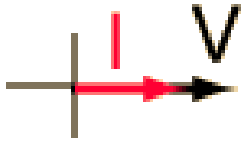
is a measure of the difficulty to pass an electric current through a conductor in DC or AC circuits



$$R = \frac{V}{I}$$

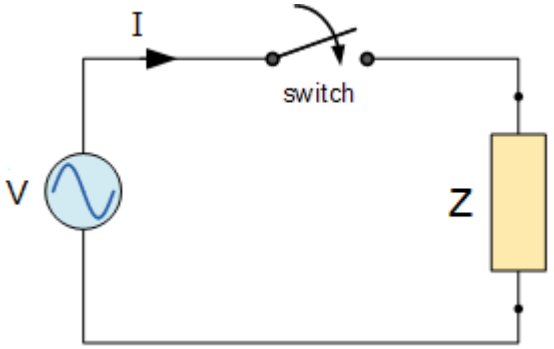


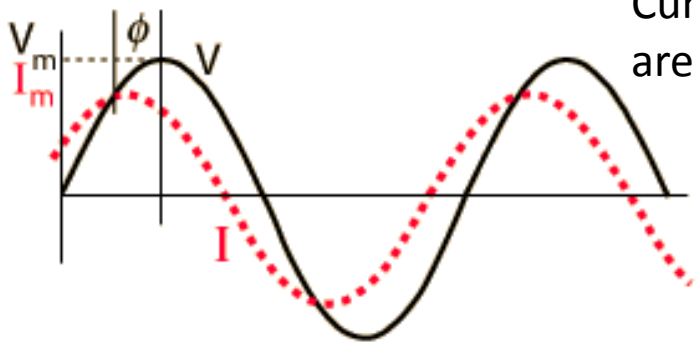
in AC circuits with the resistive load: current and voltage are <u>in phase</u>



Impedance

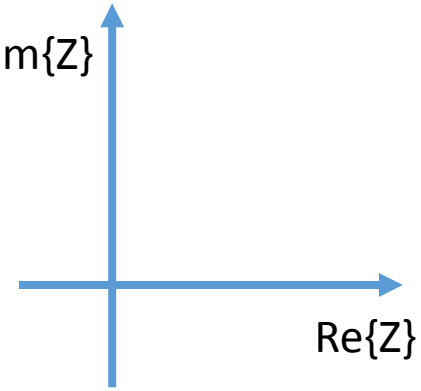
is a measure of the difficulty to pass an electric current through a conductor in AC circuits





Current and voltage are not in phase

$$Z = \frac{V}{I} = \frac{V_m e^{j2\pi ft}}{I_m e^{j(2\pi ft+\phi)}} = \frac{V_m}{I_m} e^{-j\phi} = \frac{V_m}{I_m}(cos\phi - jsin\phi) = R + jX$$

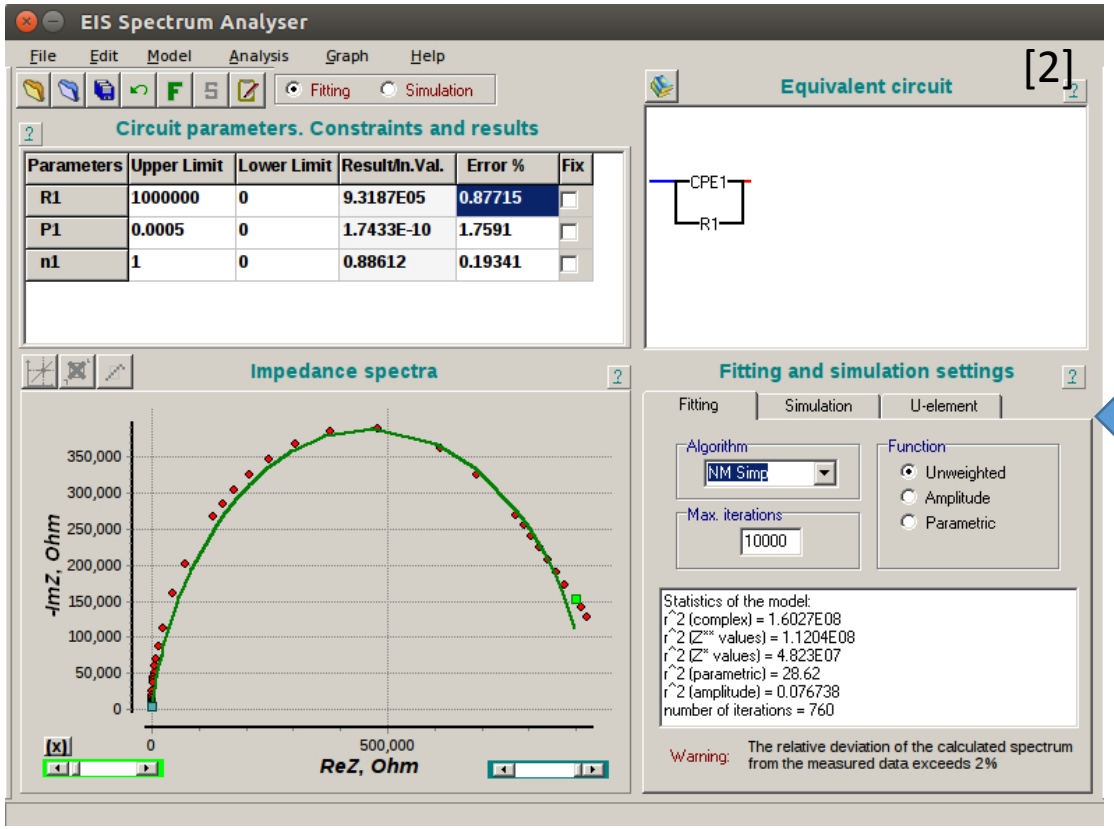


impedance is just a resistance that depends on frequency

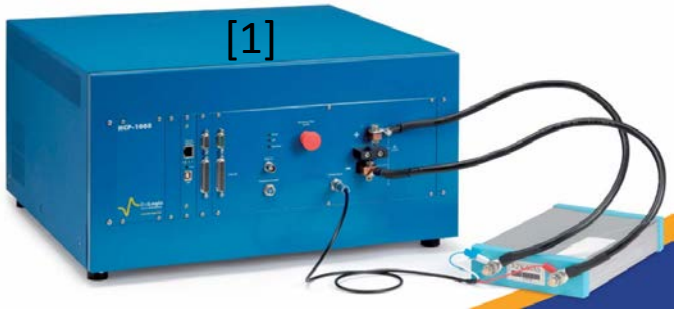
Impedance Spectroscopy

impedance measurement over a wide range of frequencies

Impedance Spectroscopy - Equipment



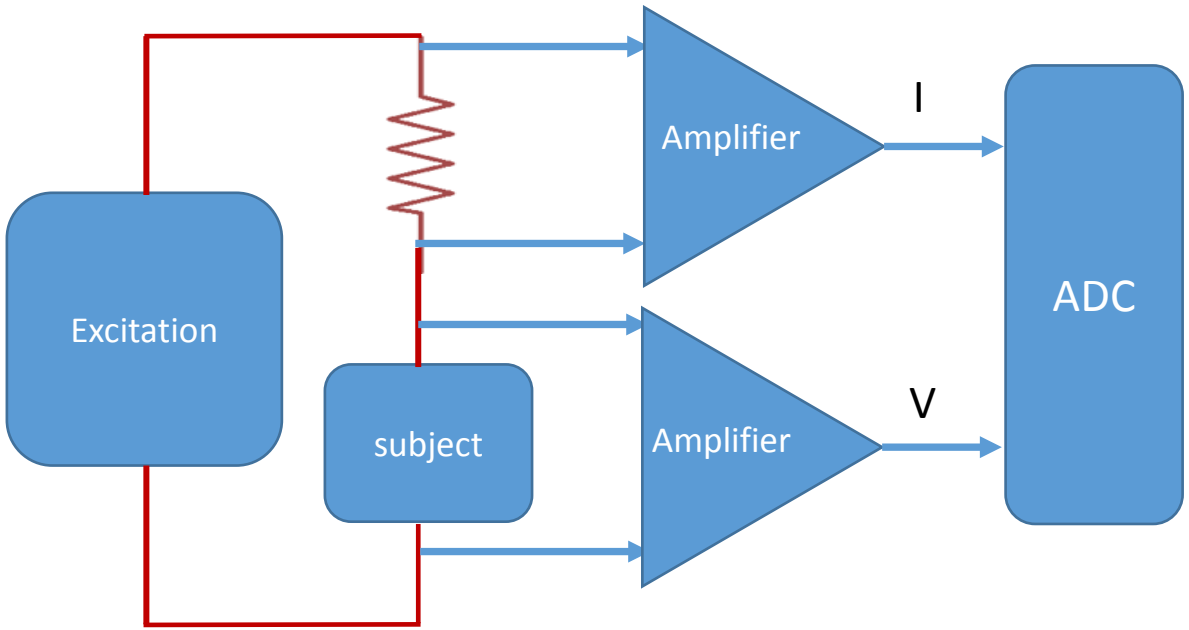
data analysis



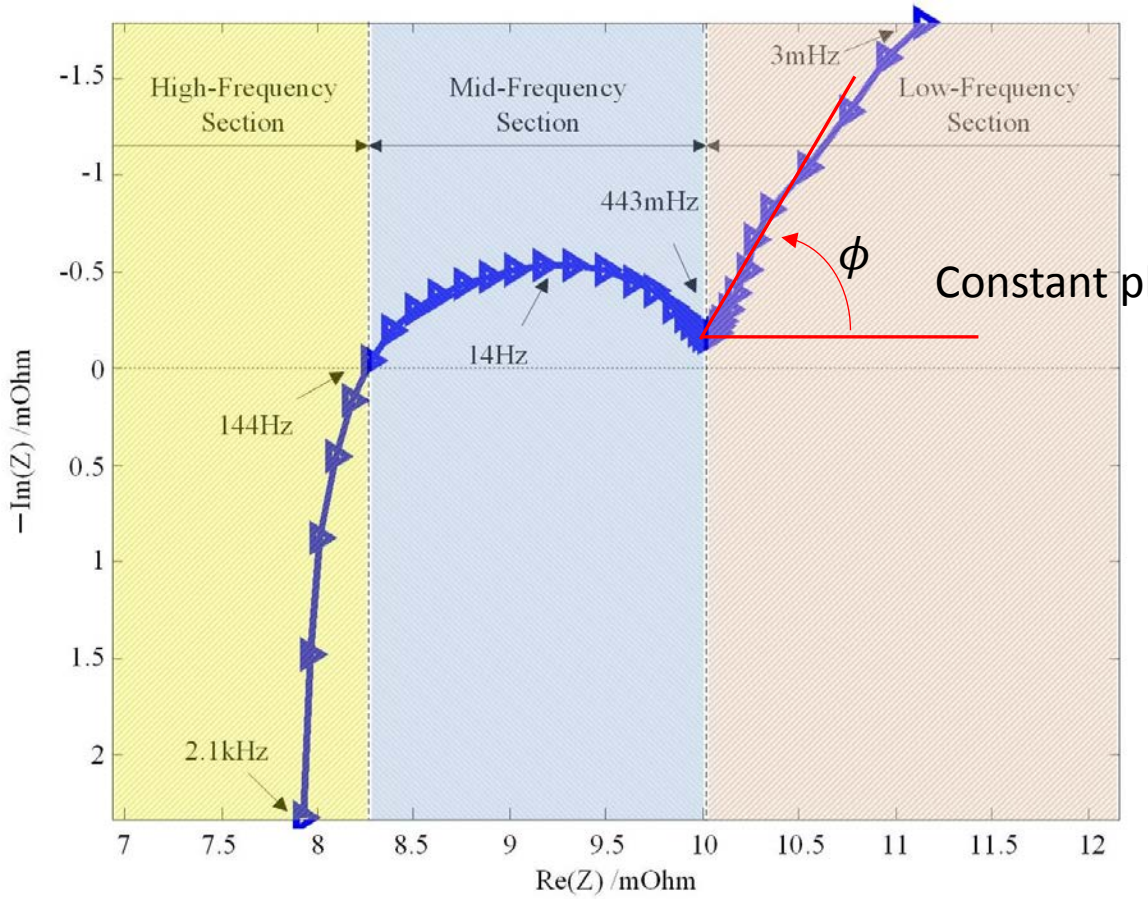
[1] HCP-1005 High current potentiostat, courtesy of BioLogic Science Instruments

[2] Potentiodynamic electrochemical impedance spectroscopy (PDEIS) software developed by the Belarusian State University.

Impedance Spectroscopy - Electronics



Typical Impedance Spectra



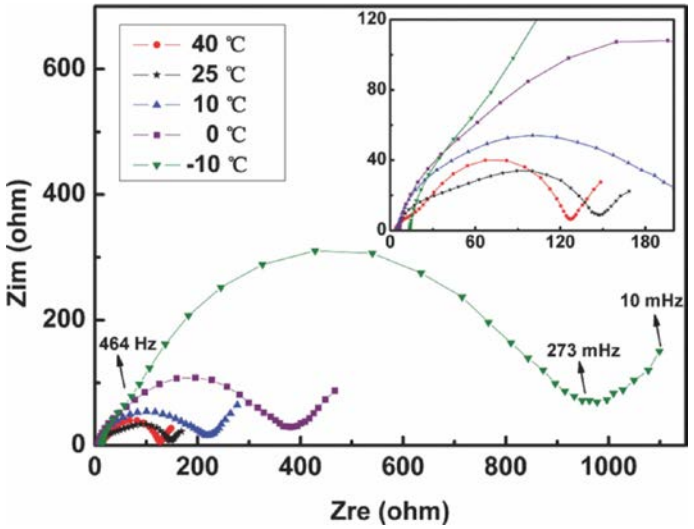
E. Barsoukov, J.R. Macdonald, *Impedance Spectroscopy – Theory, Experiment, and Applications,* Wiley-Interscience; 2 ed., 2005.

Application in Battery Energy Storage Systems

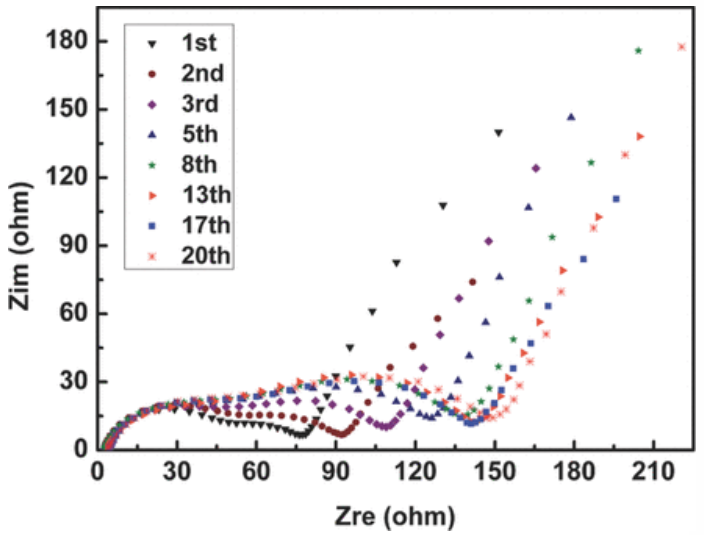
Importance of batteries

Electrochemical Impedance spectroscopy: for battery monitoring

EIS vs. Temperature

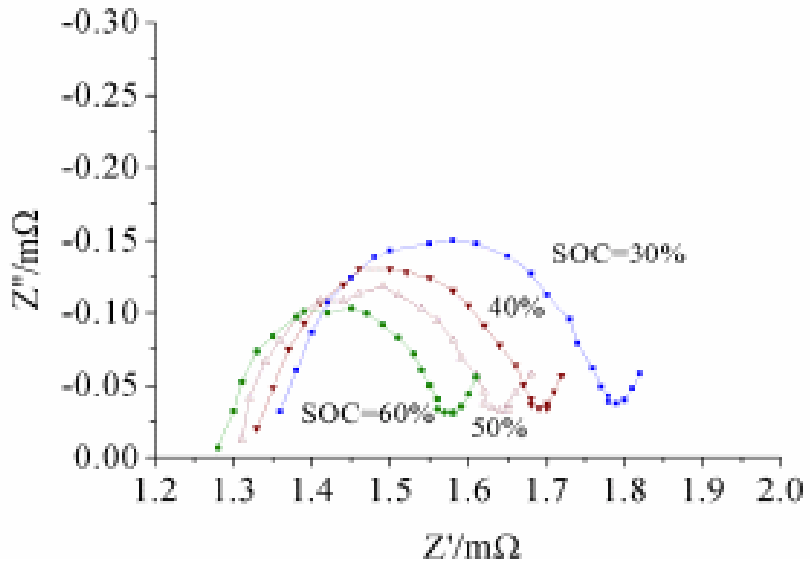


EIS vs. cycling



Li-S battery

EIS vs. state-of-charge



$LiMn_2O_4$ battery

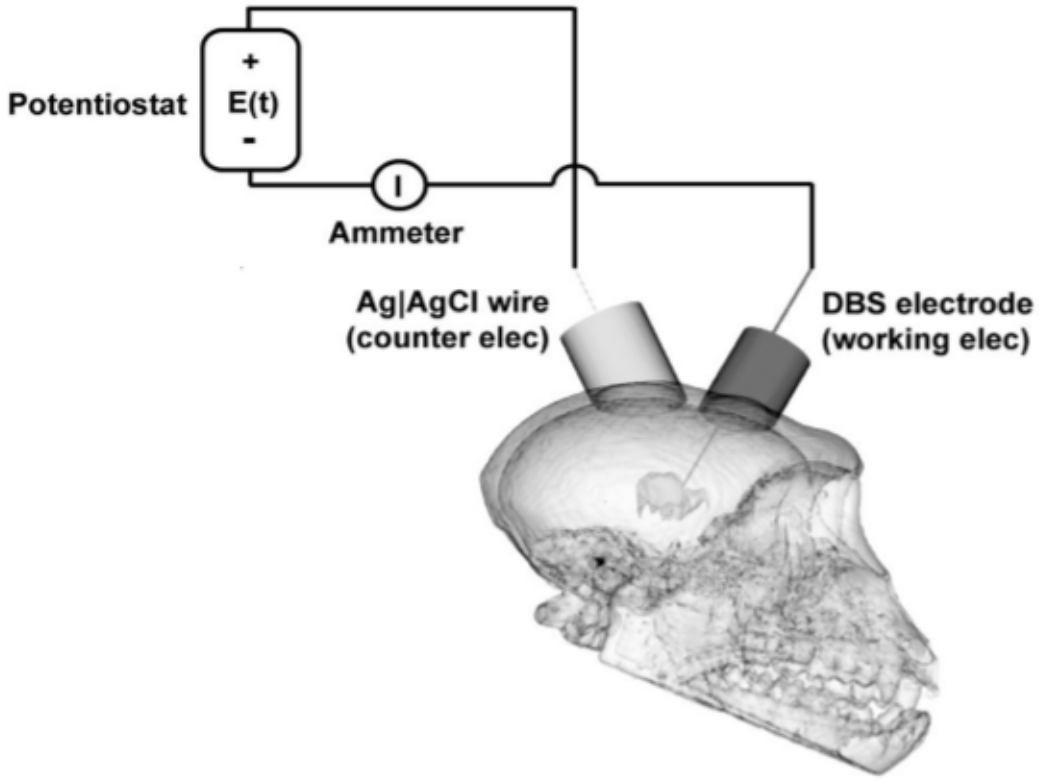
[1] Deng Z., *et al., Journal of The Electrochemical Society*, 160 (4) A553-A558 (2013).
[2] Jiang Y., *et al., Journal of Industrial Engineering and Management,* 6(2), 686-697 (2013).

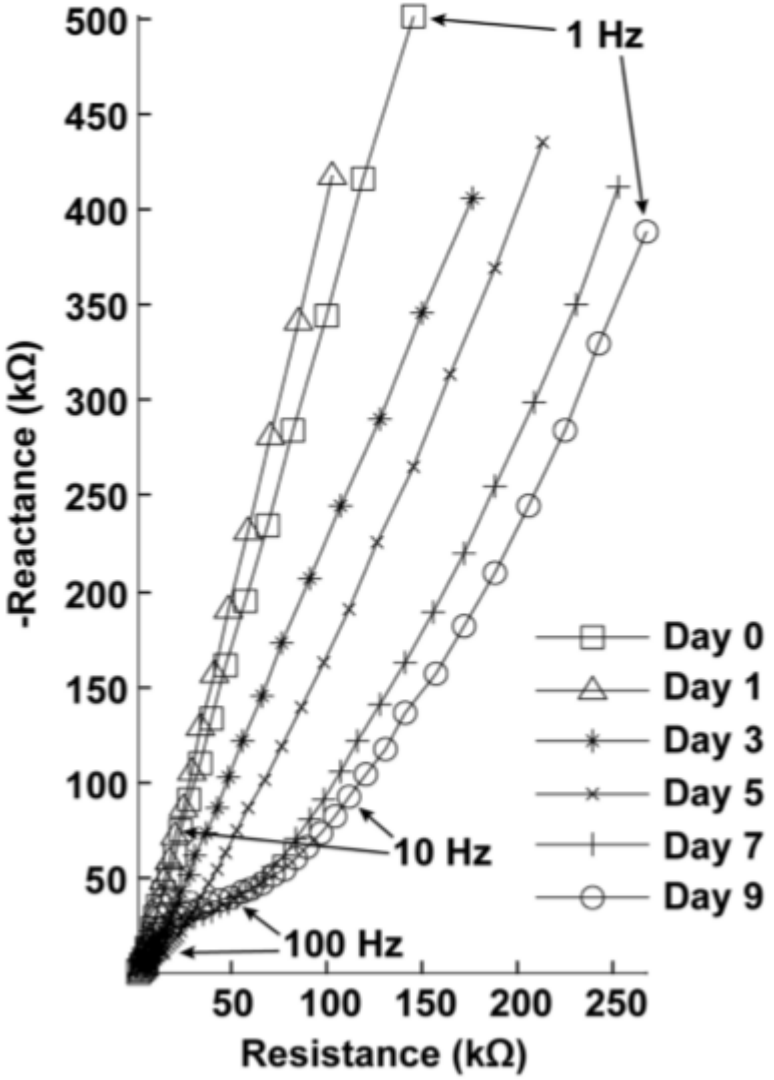
Application in Medicine

Diagnostic and Therapeutic Tools

- ❑ Lung
- ❑ Heart
- ❑ Liver
- ❑ Breast cancer
- ❑ Stroke
- ❑ Deep brain stimulation
- ❑ …

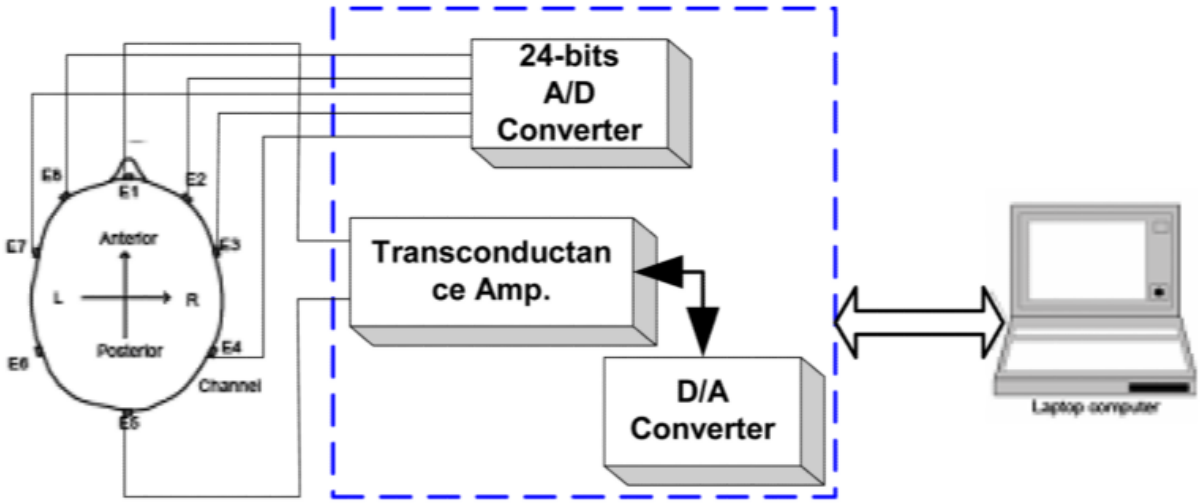
Deep Brain Stimulation

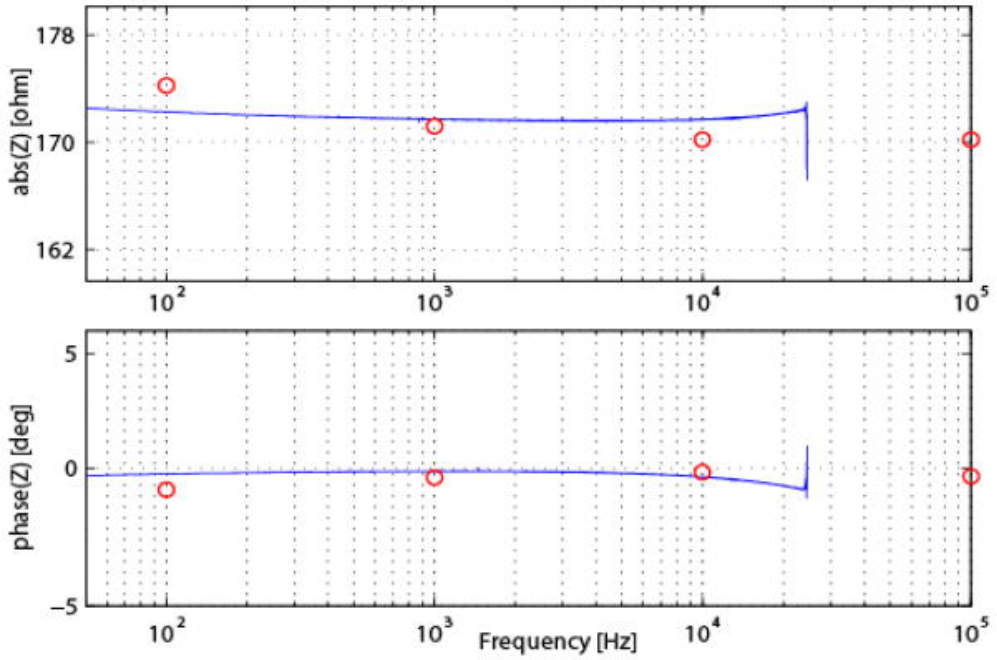




S.F. Lempka , In vivo impedance spectroscopy of deep brain stimulation electrodes, J Neural Eng. 6(4): 046001, 2009.

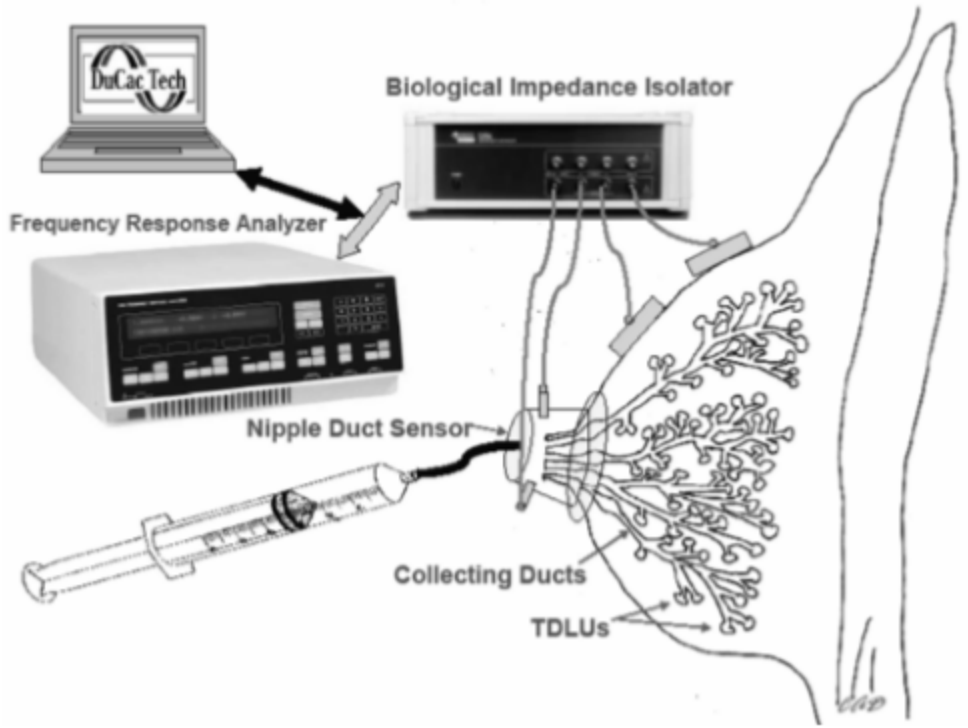
Stroke Detection

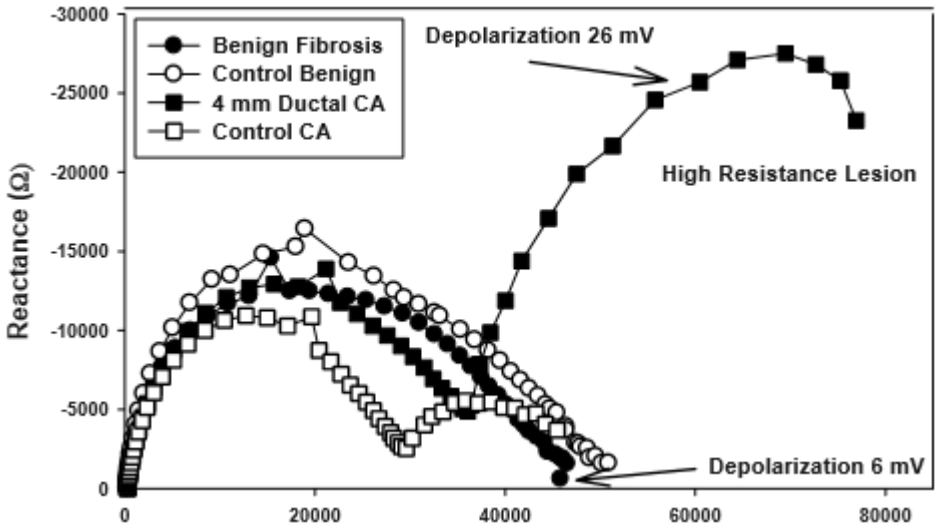




G. Bonmassar , et al., On the Measurement of Electrical Impedance Spectroscopy (EIS) of the Human Head, Int. J. Bioelectromagn., 12(1):32-46, 2010.

Breast Cancer Detection

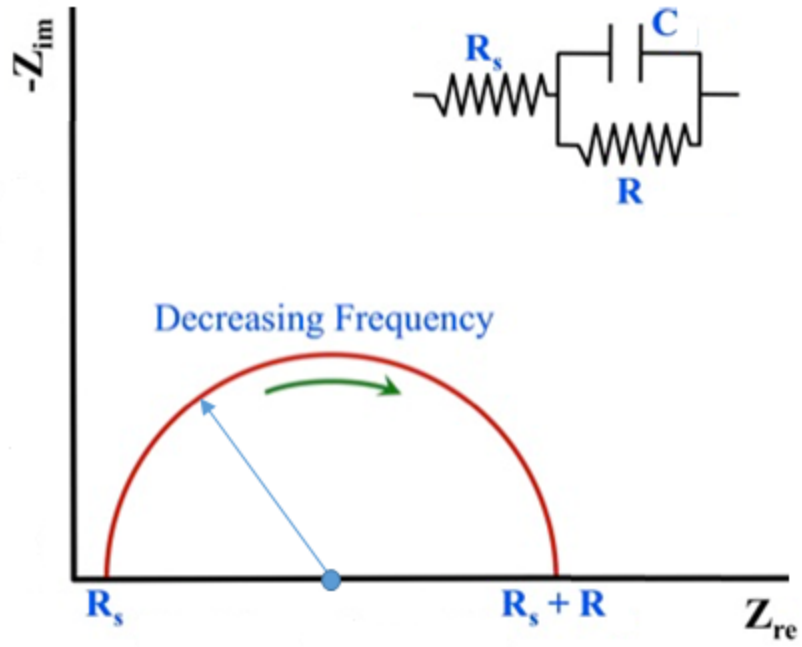




R.J. Davies et al., Diagnosis of Breast Cancer using Ductal Epithelial Impedance Spectroscopy, ICEBI 2007, IFMBE Proceedings 17, pp. 632–635, 2007

Modeling and Identification

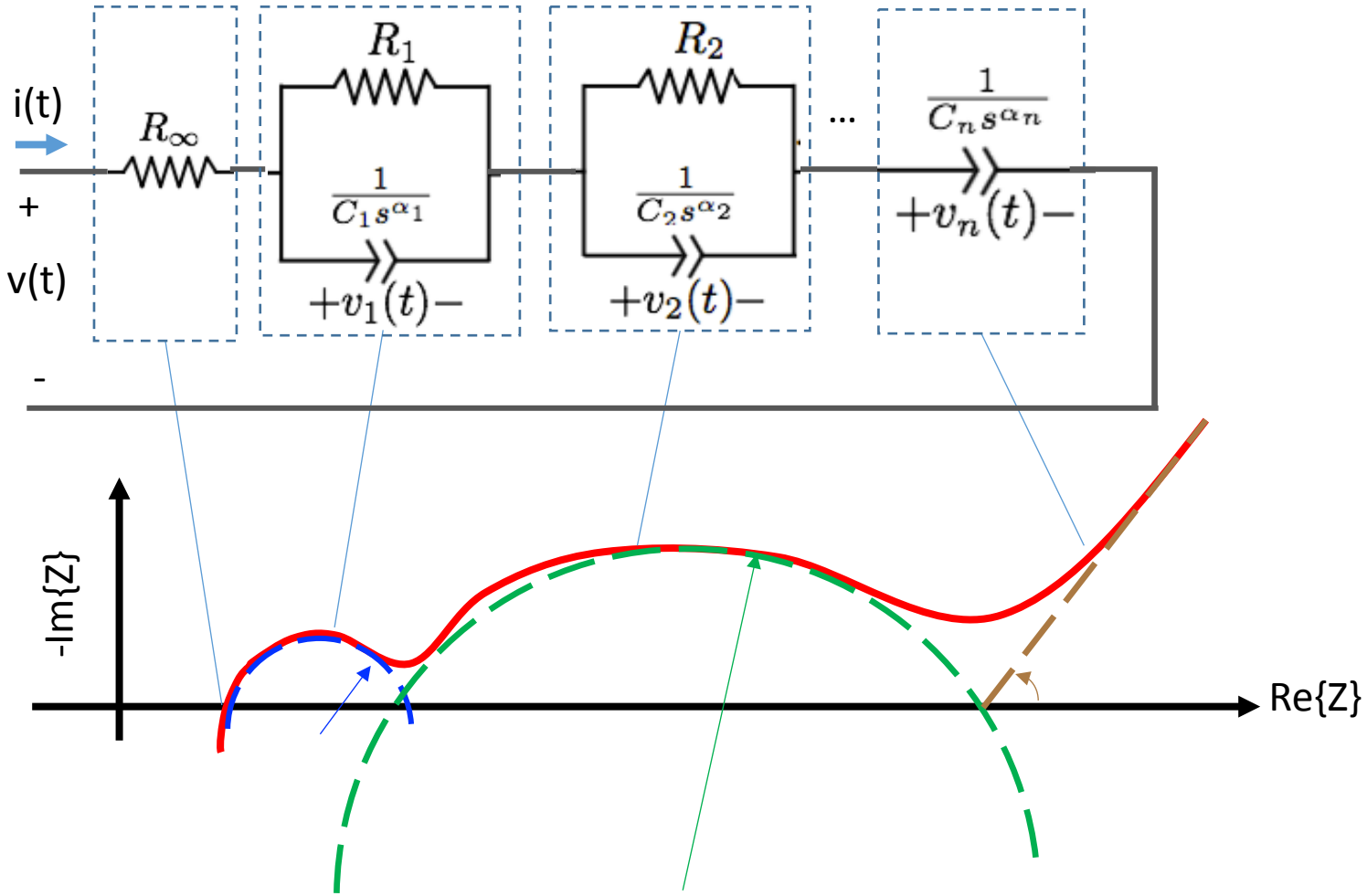
Standard Capacitor



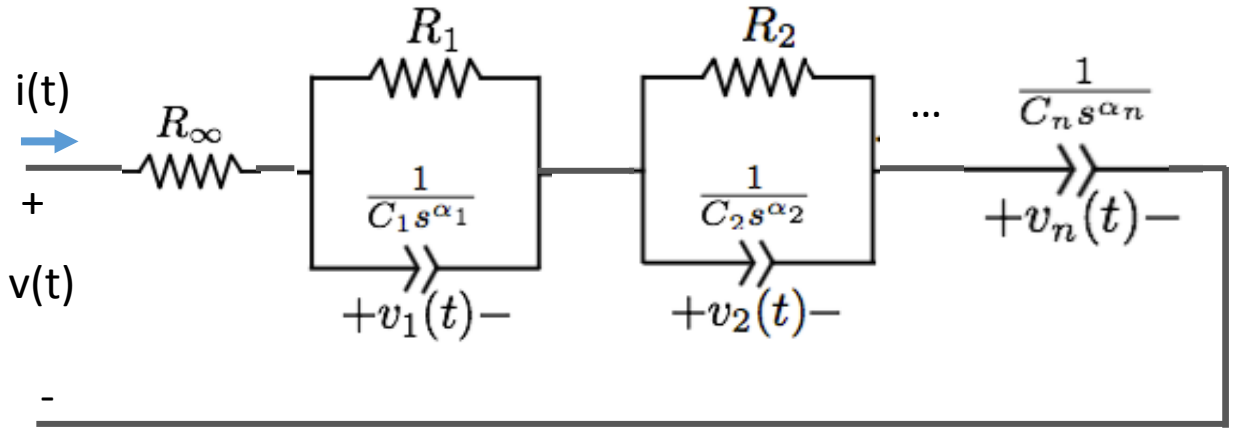
Impedance spectroscopy general model

Constant Phase Element (CPE):

$$Z_{CPE}(\omega) = \frac{1}{C_i s^{\alpha_i}}$$



Identification Problem



Given i(t) and v(t) ➡ estimate

$$\theta = [R_\infty, R_1, \cdots, R_n, C_1, \cdots, C_n, \alpha_1, \cdots, \alpha_n]$$

$$H(z,\theta) = d(\theta) + \sum_{i=1}^{n} \frac{m_i b_i(\theta) z^T}{z^{T+1} - \sum_{j=0}^{T} a_{i,j}(\theta) z^{T-j}}$$

Non-markov

$$a_{i,0}(\theta) = \alpha_i - \frac{T_s^{\alpha_i}}{R_i C_i}$$

$$a_{i,j}(\theta) = -(-1)^{j+1} \binom{\alpha_i}{j+1}$$

Binomial coeff.

$$b_i(\theta) = \frac{T_s^{\alpha_i}}{C_i}$$

$$m_i = 1$$

$$d(\theta) = R_\infty$$

$$\text{for } i = 1, \cdots, n \text{ and } j = 1, 2, \cdots, T$$

Identifiability

Definition : Let $\mathcal{M}$ be a model structure with the transfer function $T(z,\boldsymbol{\theta})$, parametrized by $\boldsymbol{\theta}$, where $\boldsymbol{\theta}$ belongs to an open subset $\mathcal{D}_T \subset \mathbb{R}^m$, and consider the equation

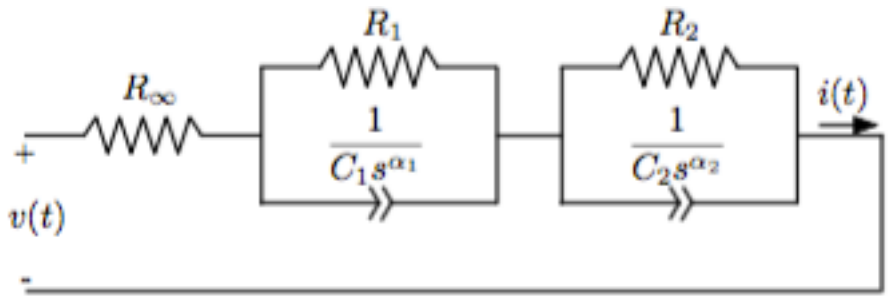
$$T(z,\boldsymbol{\theta}) = T(z,\boldsymbol{\theta}^*) \qquad \text{for almost all } z, \tag{1}$$

where $\boldsymbol{\theta},\boldsymbol{\theta}^* \in \mathcal{D}_T$. Then, the model structure $\mathcal{M}$ is said to be

- *globally identifiable* if (1) has a unique solution in $\mathcal{D}_T$,
- *locally identifiable* if (1) has a finite number of solutions in $\mathcal{D}_T$,
- *unidentifiabile* if (1) has a infinite number of solutions in $\mathcal{D}_T$.

S.M.M. Alavi, Identifiability of Generalized Randles Circuit Models, IEEE Trans. Control Systems Technology, In Press, 2017.

Identifiability of EIS model with 2-CPEs



$$\begin{cases} g_1 + g_0(T+1)\left(\dfrac{1}{\alpha_1 - T} + \dfrac{1}{\alpha_2 - T}\right) = 0 \\ g_2 - g_0(T+1)(\hat{a} + \hat{b} + \hat{c}) = 0, \end{cases}$$

$$g_2(\theta) = a_{1,T}a_{2,T-2} + a_{1,T-1}a_{2,T-1} + a_{1,T-2}a_{2,T}$$

$$g_1(\theta) = a_{1,T}a_{2,T-1} + a_{1,T-1}a_{2,T}$$

$$g_0(\theta) = a_{1,T}a_{2,T}.$$

$$a_{i,j+1} = -\frac{(\alpha_i - j - 1)}{j+2}a_{i,j}$$

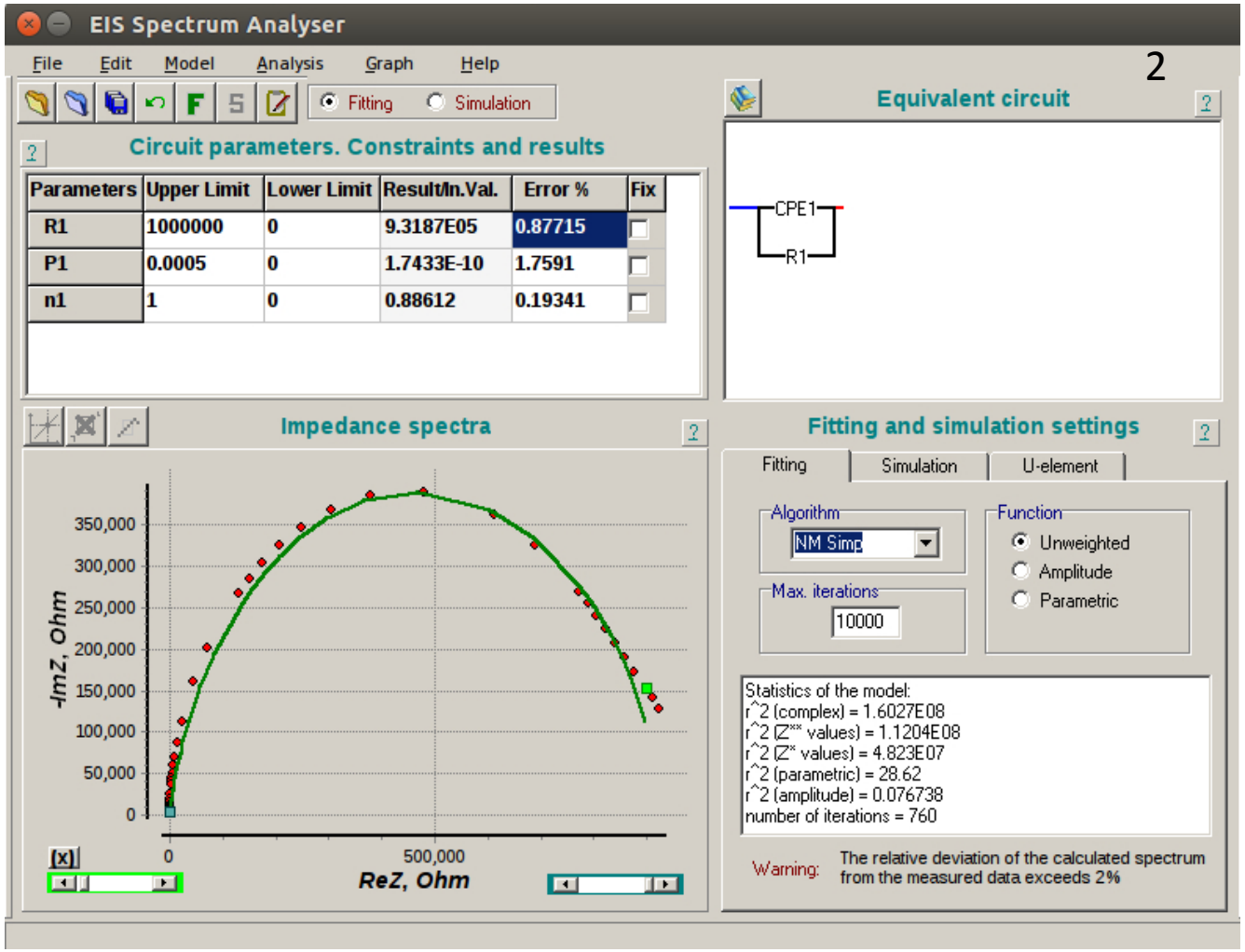
$$\hat{a} = \frac{T}{(\alpha_2 - T)(\alpha_2 - T + 1)}$$

$$\hat{b} = \frac{(T+1)}{(\alpha_1 - T)(\alpha_2 - T)}$$

$$\hat{c} = \frac{T}{(\alpha_1 - T)(\alpha_1 - T + 1)}$$

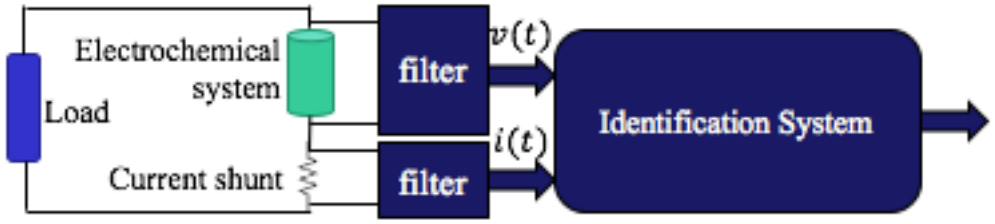
S.M.M. Alavi, et al. Structural Identifiability Analysis of Fractional Order Models with Applications in Battery Systems, arXiv:1511.01402

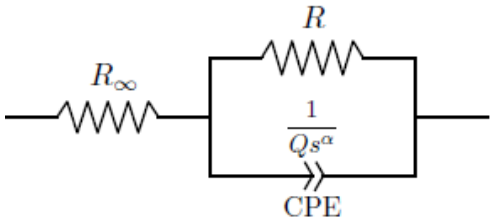
Frequency domain fitting



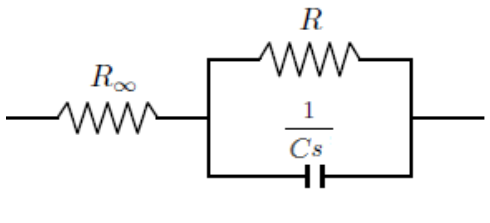
Potentiodynamic electrochemical impedance spectroscopy (PDEIS) software developed by the Belarusian State University.

Time domain fitting (1-CPE globally ident.)

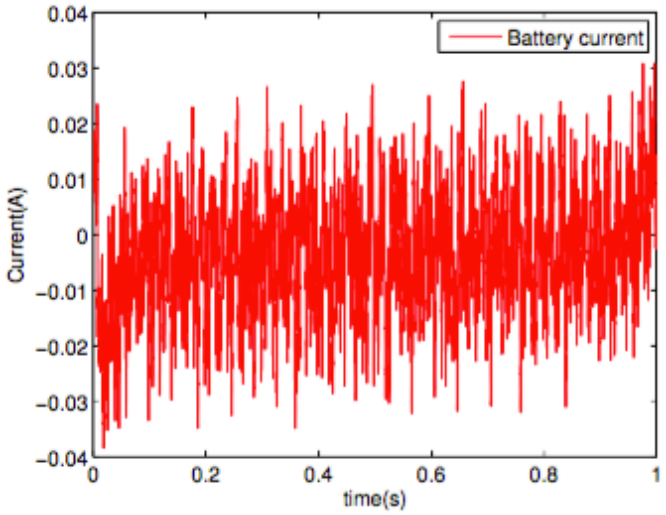


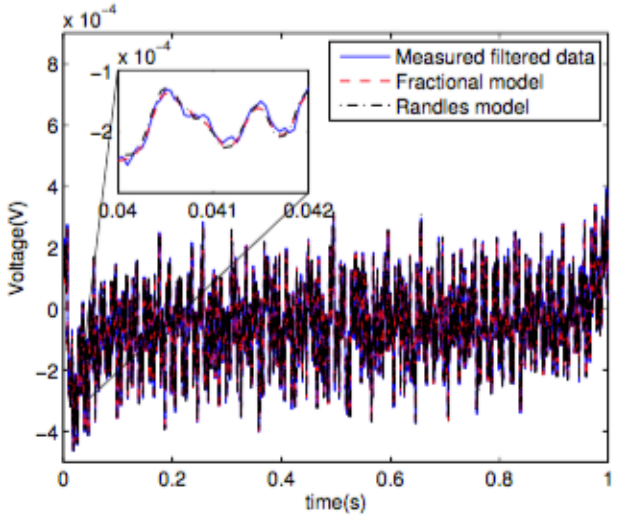


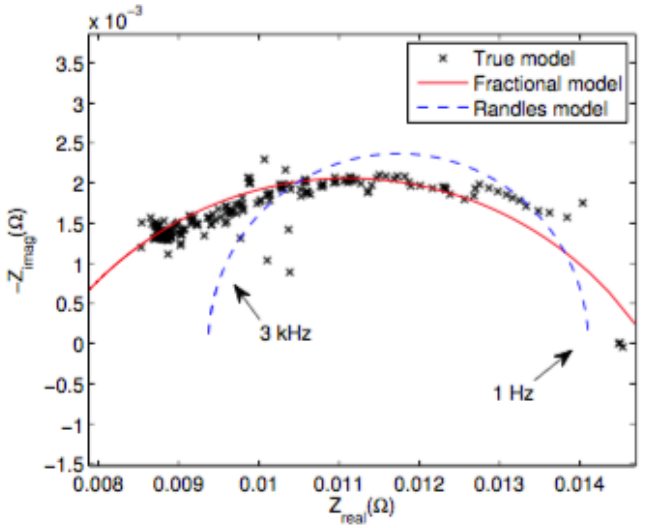
EIS 1-CPE fractional model



Randles model

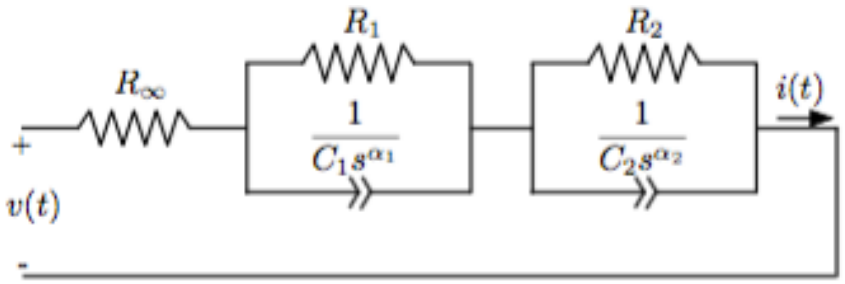


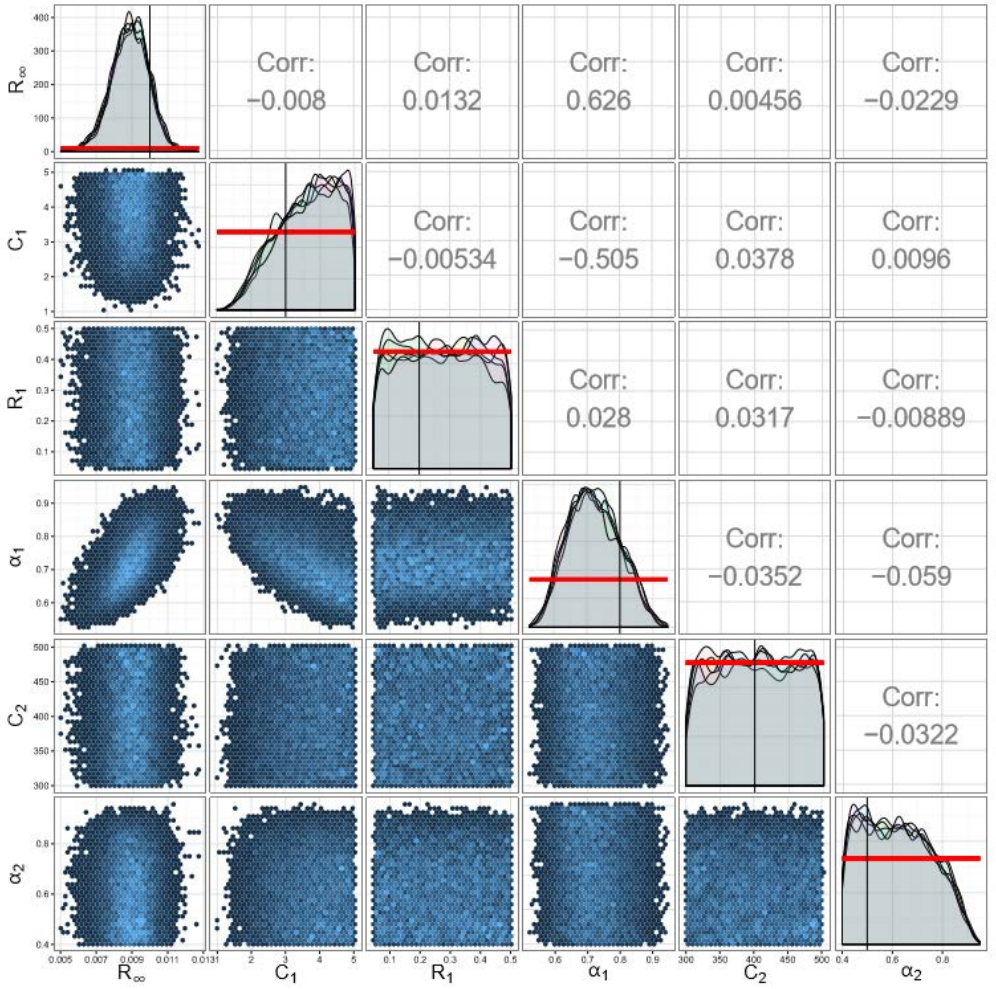




SMM Alavi, et al., Time-domain fitting of battery electrochemical impedance models, Journal of Power Sources 288, 345-352, 2015

Time domain fitting (2-CPE locally ident.)





PE Jacob, **SMM Alavi**, A Mahdi, SJ Payne, DA Howey , Bayesian Inference in Non-Markovian State-Space Models With Applications to Battery Fractional-Order Systems, *IEEE Transactions on Control Systems Technology*, In Press, 2017

Conclusions

- Impedance Spectroscopy is an important tool for modeling, diagnostics, …
- It’s medical application is vast, but each application may require its own hardware and software!
- Time domain identification of EIS models is still of open problems.